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A NOTE ON HOMOGENIETY AND NATIONAL INCOMES

A. Qadir

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By

A. Qadir*

Abstract

There is a fundamental difference between the Walrasian and Metzleric systems in that the Walrasian system is homogeneous while the Metzleric system is not. A third possibility is of a system which is not completely homogeneous, but may be regarded as being approximately homogeneous. We show that the price system and national incomes system are both of this type - the only difference being the degree to which the approximation holds. We then develop propositions intermediate between the Walrasian and Metzleric systems which would hold for the actual price system and national income system. We find that the so-called gravity model is suitable (approximately) for this system.

*Mathematics Department, Islamabad University, Islamabad, Pakistan.

S. 1. Introduction

Mundell, in his chapter on Comparative Statics and Homogeniety [4] states three propositions for the Walrasian system [1] and three weaker propositions for the Metzleric [2,3] system. The difference between the sets of propositions is, that in the Walrasian system shifts in all prices due to shifts in given prices have a given range, while in the Metzleric system there is no range. The difference between the two systems is that the Walrasian system is homogeneous while the Metzleric system is not.

There would be an intermediate system which is homogeneous upto some degree of accuracy in the relevant variables (prices or national incomes). Then, to the accuracy given we could prove the Walrasian propositions. This means that the range mentioned in the propositions would not be given exactly, but only approximately. Of course, the greater the inaccuracy in the homogeniety, the greater would be the uncertainty in the predicted range.

This system has various advantages over both systems considered earlier:-

1. It makes stronger predictions than the Metzleric system.
2. It is valid for all types of variables, while the Walrasian system is unlikely to be valid for any.
3. It removes the difference between open and closed ranges which comes in the Walrasian system. Clearly this difference could not be realistically maintained.

4. As we shall see later the Walrasian system is not valid for prices, but is merely a good approximation.

5. As we shall see later it enables the derivation of the socalled "gravity model" for international trade as a full-fledged theory (up to a degree of accuracy).

This system gives us the ability, so to speak, to have our cake (as in the Metzleric system) and eat it too (as in the Walrasian system).

§. 2. The New System

We shall now construct the new system, which lies between the Metzleric and Walrasian systems. In this system we shall not be making the assumption of homogeniety, nor shall we be leaving it out altogether. Instead, we shall assume that the assumption of homogeniety is valid for the system of excess demands $x_i(x_j)=0$, up to an uncertainty u in the variables under consideration (x_1) (over the index set I). By this we mean that to each x_1 there corresponds an $\bar{x}_1$ which is approximately equal to x_1 up to accuracy u, i.e. statiscally speaking

$$\bar{x}_i = x_i \pm u \qquad (2.1)$$

such that the system is homogeneous of degree 0 in $\bar{x}_i$.

Let us take the system to have randomly defined variables. We can then take

$$\sum_{i=1}^{n} x_i = 1$$

and for large n

$$x_i \ll 1$$

$$\bar{x}_i = \bar{x}_j \qquad \forall i, j \quad I$$
$$= \bar{x} \quad \text{(say)}$$

Then we see that u would be the standard deviation given by

$$u = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}} \qquad (2.5)$$

It is obvious that this statistical analysis can hold only for large n, and that as $n \longrightarrow \infty$ we would get $u \longrightarrow 0$ and hence the system would be homogeneous, Equally, for small n, we see that the uncertainty must become about as large as the predictions being made. Thus for large n we would get the Walrasian system and for small n the Metzleric system. For general n we should have this intermediate system.

It is obvious that the propositions for this system corresponding to those for the Walrasian and Metzleric systems [4] will be modified forms of the previous propositions, being different in giving a range of accuracy to the predictions.

1. A shift from the numeraire to member i of the index set causes a rise in the variable x_i, dx_i, and a rise in all other variables x_i, dx_j, such that

$$dx_j \leq dx_i \pm u \qquad (2.6)$$

2. A shift from member if of the index set to member jo of the index set causes a rise in the variable x_i, dx_i, and a fall in the variable x_k, dx_k, such that

$$dx_k \in (dx_j, dx_i) \pm u \qquad (2.7)$$

3. A shift from all members of the index set, to member j of the index set, causes a rise in the variable x_i, dx_i, and changes all other variables x_j by dx_j such that

$$\frac{dx_j}{x_j} \leq \frac{dx_i}{x_i} \pm 2u \qquad (2.8)$$

Here, we can obviously select naumeraire member arbitrarilly. The proof will obviously follow on the lines given for the Walrasian system in reference 4, except that it would hold for the system with variables $\bar{x}_i$ rather than x_i. Finally, we can replace $\bar{x}_i$ by $x_i \pm u$. Clearly if u is large these propositions reduce to the Metzleric propositions. Similarly, if u is zero these propositions reduce to the Walrasian system. However, in the general case they allow stronger predictions to be made than can be done in the comparatively realistic Metzleric system though weaker than can be made in the idealized Walrasian system.

S.3. The Systems of Prices and National Incomes.

It is generally assumed that the price system is hologeneous, i.e. if we instantaneously scale all prices, p_i, by an arbitrary factor, there will be no change in the economy, E, i.e. for n commodities

$$\left.\begin{aligned} p_i &\rightarrowtail \tilde{p}_i = k\,p_i \quad i = 1, ---, n \\ E &\rightarrowtail \tilde{E} = E \end{aligned}\right) \qquad (3.1)$$

However, it is assumed that the national income system (represented by $\{y_i\}_{i \in I}$) is inhomogeneous, ie.e

$$\left.\begin{array}{ll} y_i \longrightarrow \tilde{y}_i = ky_i & i = 1, \text{---}, n \\ E \longrightarrow \tilde{E} \neq E & \end{array}\right\} \quad (3.2)$$

Now prices may be scaled instantaneously either by a change in the unit of the commodities or by a change in the value of the unit of currency or by a sudden change in the demand or supply. Clearly there can be no immediate economic effect by a change of units, as that is not an economic change. However, a change in the demand or supply would have an economic effect. This may be most easily seen for a small number of commodities, a doubling of prices of luxuries and necessities will cause a shift in demand from the luxuries to the necessities.

Similarly, national incomes may be scaled by a change in the unit of currency in which the incomes are being measured or by a change in the amount of trade. Clearly the former will cause no change in the international economy while the latter will.

The reason why the national income system is more obviously inhomogenious than the price system is that there are few countries being considered and condition (2.3) is not satisfied, so that we can not generally take

$$y_i \simeq y_j \quad \forall i, j \in I \qquad (3.3)$$

However, we could still write (for sufficiently large u)

$$y_i \simeq y_j \pm u \quad \forall i,j \in I \qquad (3.4)$$

As in the case of prices, we would take the largest y_i to be the numeraire or basic national income. (In the price system we usually take basic commodities as a group as the numeraire.) In that case the analysis will still hold good provided that for all other y_i, eqn. (3.4) will hold for a u sufficiently small to make meaningful predictions (2.6) - (2.8).

§. 4. Conclusion

We see that both the price system and the national income system would fit into the new system given in section 2, and not into the Walrasian system under genuine economic scalings, but would fit into the Walrasian system under changes of units. Taking the economic scalings, they would both certainly fit into the Metzleric system, but the new system gives stronger predictions.

Notice that we could derive the so-called gravity model of international trade for a system of trade where the condition (2.3) holds for national income. If we take f_i to be the total trade of country i divided by the total world trade

$$\sum_{i=I}^{n} f_i = 1$$

Now the trade between countries i and j is the trade of country i, f_i, and no other, ($1-f_i$), multiplied by the trade of country j, f_j, and no other, ($1-f_j$),

$$f_{ij} = f_i (1 - f_i) f_i (1- f_i)$$
$$= f_i f_j - f_i^2 f_j - f_i f_j^2 + f_i^2 f_j^2 \qquad (4.2)$$

Now condition (2.3) means (taken with eqn. (4.1) that

$$f_i \ll 1 \qquad \forall i \in I \qquad (4.3)$$

$$f_{ij} \approx f_i f_j \qquad (4.4)$$

having taken f_i as the ratio of the national income of country i to the world income, in eqn. (2.3) with x_i replaced by f_i.

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